$\mathbb{C} = \{(a,b)\}$ $a, b \in \mathbb{R}$

нека $(a_1 b_1), (a_2 b_2) \in \mathbb{C}$

Дефиниране операцията събиране на комплексни числа

$$(a_1, b_1) + (a_2 b_2) = (a_1 + a_2)(b_1 + b_2)$$

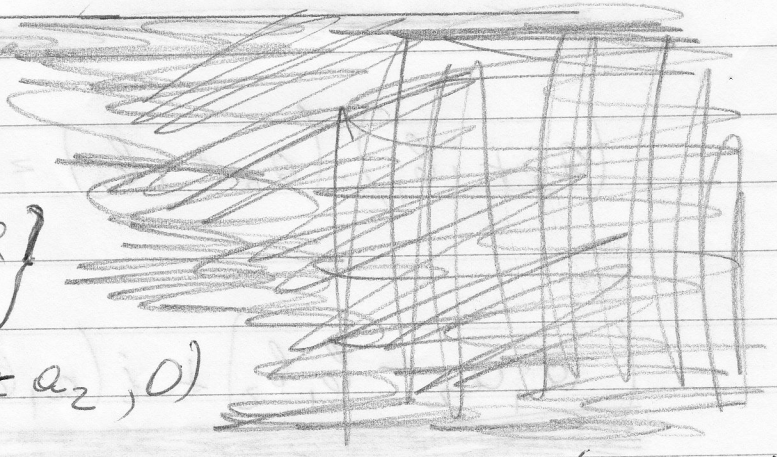
Дефиниране операцията умножение на комплексни числа

$$(a_1, b_1) . (a_2 b_2) = (a_1 a_2 - b_1 b_2, a_1 b_2 + b_1 a_2)$$

$$(a_1 b_1) = (a_2 b_2) \iff a_1 = a_2, \; b_1 = b_2.$$

разгл множеството

$$\{(a, 0) \mid a \in \mathbb{R}\}$$



събиране $(a_1, 0) + (a_2 0) = (a_1 + a_2, 0)$

умножение $(a_1, 0)(a_2 0) = a_1 a_2 - 0.0, \; a_1 0 + a_2 0 = (a_1 a_2, 0)$

$$(0,1) = i$$

$$(0;1)(0,1) = (0\cdot 0 - 1\cdot 1), (0\cdot 1 + 1\cdot 0) = (-1;0) \Rightarrow$$

$$\underline{\underline{i^2 = (0,1)^2 = -1}}$$ - имагинарна единица

Разлагане на множество

$$(a,b) = (a,0)(0,b)$$

$$b\cdot i = (b;0)(0,1) = (b\cdot 0 - 0\cdot 1), (b\cdot 1 + 0\cdot 0) = (0,b)$$

$$\underline{\underline{(a,0) + (0,1)(b,0) = a + ib}}$$

$$\underline{\underline{(a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i(b_1 + b_2)}}$$

$$(a_1 + ib_1)\cdot(a_2 + ib_2) = (a_1 a_2 + \underbrace{b_1 b_2 i^2}_{-b_1 b_2} + \underbrace{a_1 b_2 i + a_2 b_1 i}_{\updownarrow}) =$$

$$\underline{\underline{(a_1 a_2 - b_1 b_2) + i(a_1 b_2 + b_1 a_2)}}$$

Нека $z = a + bi \in \mathbb{C}$

a - реална част на z

b - имагинерна част на z

$$b = \mathrm{Im}\, z$$

Нека $z = a + bi \in \mathbb{C}$

$\bar{z} = a - bi$ - комплексно спрегнато на z

$$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$$

Нека $\left.\begin{array}{l} z_1 = (a+bi) \\ z_2 = (c+di) \end{array}\right\} \Rightarrow$

$$z_1 + z_2 = (a+bi) + (c+di) = (a+c) + i(b+d)$$

Нека $\left.\begin{array}{l} z = a + bi \in \mathbb{C} \\ \bar{z} = a - bi \end{array}\right\} \Rightarrow$

$z + \bar{z} = 2a$ - реално число

$z \cdot \bar{z} = (a+bi)(a-bi) = a^2 - i^2 b^2 \Rightarrow$

$$z \cdot \bar{z} = a^2 + b^2$$

модул

$|z| = \sqrt{z\bar{z}} = \sqrt{a^2+b^2} \geq 0$

$-z$ - противоположно на z

ако $z = a+bi$ и $z_1 = \frac{\bar{z}}{|z|^2}$ то

$$z \cdot z_1 = z \frac{\bar{z}}{|z|^2} = (a+bi)\frac{(a-bi)}{(\sqrt{a^2+b^2})^2} =$$

$$= \frac{a^2 - (i^2)b^2}{a^2+b^2} = \frac{a^2+b^2}{a^2+b^2} = 1$$

$(1-i)x + (2-i)y = 5-3i$

$x - xi + 2y - yi = 5-3i$

$x+2y + i(-x-y) = 5-3i$

$$\begin{cases} x+2y = 5 \\ i(-x-y) = -3i \end{cases} \Rightarrow -x-y = -3 \mid \cdot(-1) \Rightarrow x+y = 3$$

изваждаме

$x+2y - x - y = 5-3 \Rightarrow y = 2$ и $x = 1$

База на имагинарното число

степен - n	$n=1$	$n=2$	$n=3$	$n=4$
	i	-1	$-i$	1

↑

или

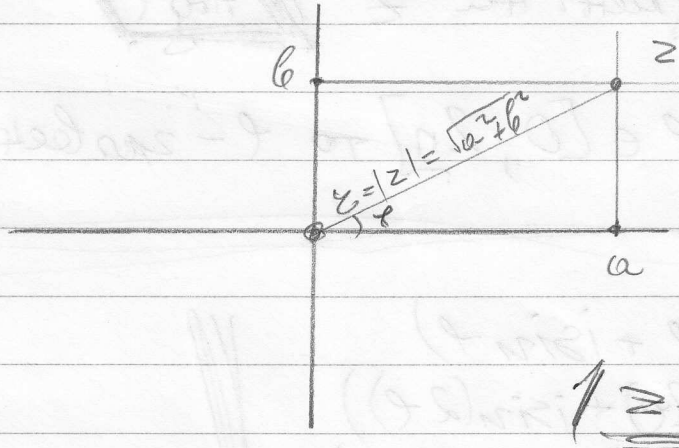
$$i^0=1;\ i^1=i;\ i^{-2}=-1;\ i^3=-i;\ i^4=1$$

$k \in R$

1. ако степента се дели на 4 то n_k
2. ако степента при делене с 4 дава остатък 1 то $\Rightarrow i^n=i$
3. ако степента при $-n-$ дава остатък 2 то $\Rightarrow$ $i^n=-1$
4. ако степента при $-n-$ дава остатък 3 то $\Rightarrow$ $i^n=-i$

Тригонометричен вид на комплексните числа

Нека $z=a+ib \in \mathbb{C}$



$$z=r(\cos\varphi+i\sin\varphi)$$

$a=r\cos\varphi$

$b=r\sin\varphi$ и $ib=ir\sin\varphi$

Нека

$$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1)$$
$$z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$$

то:

$$z_1 \cdot z_2 = r_1 r_2(\cos\varphi_1\cos\varphi_2 - \sin\varphi_1\sin\varphi_2) +$$
$$+ i(\cos\varphi_1\sin\varphi_2 + \sin\varphi_1\cos\varphi_2) =$$
$$= r_1 r_2(\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2))$$

$$\frac{z_1}{z_2} = \frac{r_1(\cos\varphi_1 + i\sin\varphi_1)}{r_2(\cos\varphi_2 + i\sin\varphi_2)} = \frac{r_1}{r_2}(\cos(\varphi_1-\varphi_2) + i(\sin\varphi_1-\varphi_2))$$

Нека $z = r(\cos\varphi + \sin\varphi)$

φ - аргумент на z $\boxed{\text{Arg}\,\varphi}$

ако $\varphi \in [0; 2\pi]$ то φ - главен аргумент

$$z = r(\cos\varphi + i\sin\varphi)$$
$$z^2 = r^2(\cos(2\varphi) + i\sin(2\varphi))$$
$$z^3 = r^3(\cos(3\varphi) + i\sin(3\varphi))$$
$$z^n = r^n(\cos(n\varphi) + i\sin(n\varphi))$$

формула на Моавър

$$\varepsilon_k = \sqrt[n]{z}\left(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right) \quad \text{при } k = 0,1,\ldots$$

нека

$$\varepsilon_k^n = \left(\sqrt[n]{z}\right)^n\left(\cos\frac{(\varphi+2k\pi)}{n}\cdot n + i\sin\frac{(\varphi+2k\pi)}{n}n\right) \Rightarrow$$

$$\varepsilon_k^n = z(\cos\varphi + i\sin\varphi)$$

нека

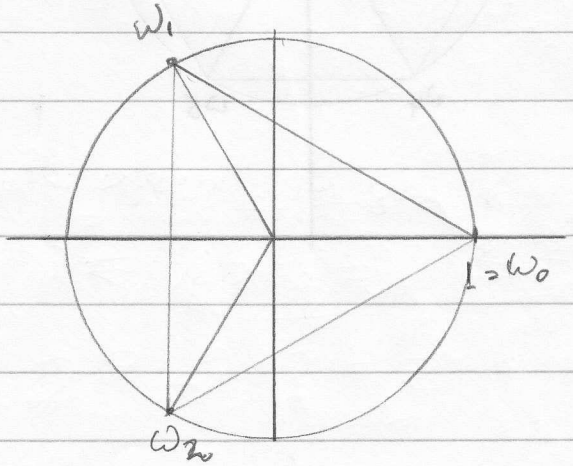
$$\omega_k = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$$

при $k = 0,1,\ldots n-1$

при $n = 3$

$$\omega_0 = \cos 0 + i\sin 0 = 1$$
$$\omega_1 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$$
$$\omega_2 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$$

и си остава n
само ни се проверя
спрямо n



при $n = 6$

$w_0 = 1$

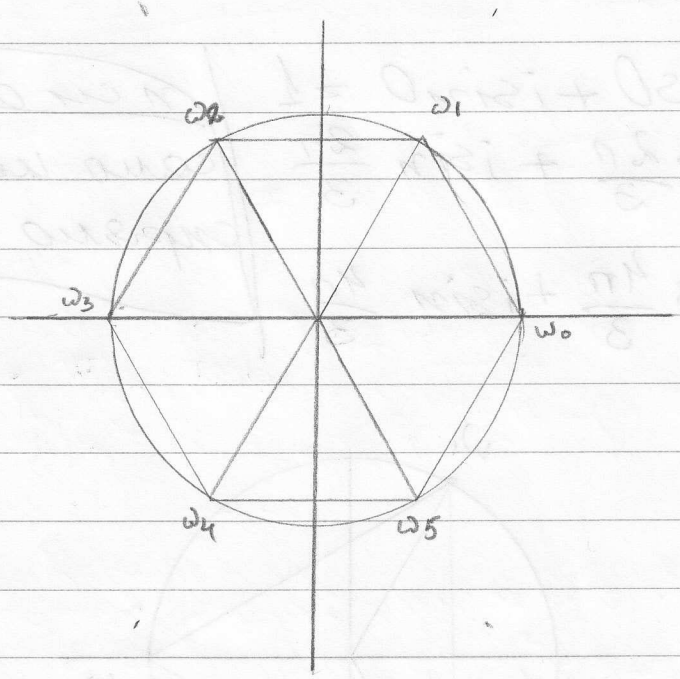
$w_1 = \cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

$w_2 = \cos\frac{2\cdot 2\pi}{6} + i\sin\frac{2\cdot 2\cdot\pi}{6} = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$w_3 = \cos\frac{2\cdot 3\cdot\pi}{6} + i\sin\frac{2\cdot 3\cdot\pi}{6} = \cos\pi + \sin\pi$

$w_4 = \cos\frac{2\cdot 4\pi}{6} + i\sin\frac{2\cdot 4\pi}{6} = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

$w_5 = \cos\frac{2\cdot 5\pi}{6} + i\sin\frac{2\cdot 5\pi}{6} = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$



Задача

Търси се

модул ($|z|$) и комплексно на $\operatorname{Arg} t$

$z = 1 \Rightarrow R = |z| = \sqrt{1} = 1$ модул

$1 = |z|(\cos t + i \sin t)$ но $|z| = 1$ т.е.

$\cos t + i \sin t = ?$ или

$(1;0) \Rightarrow$ при $t = 0$

$z = 1 - i\sqrt{3}$ т.е. $|z| = \sqrt{1+3} = 2$

$z = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) \Rightarrow$

$\cos t = 1/2$ и $\sin t = -\sqrt{3}/2$

$\arg z = \frac{5\pi}{3}$

$$\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3k+2} = 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^{3k+2} =$$

$$= \left(\cos\frac{(3k+2)2\pi}{3} + i\sin\frac{(3k+2)2\pi}{3}\right) =$$

$$= \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\frac{1}{2}$$